Moving, comparing, transforming graphs: A bodily approach to functions

MOVING, COMPARING, TRANSFORMING GRAPHS: A BODILY APPROACH TO FUNCTIONS

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The workshop aims at presenting and discussing activities in which graphing motion technology named WiiGraph is used. The activities offer possible lines of (inter)action within the classroom to introduce discourses about the concept of function, graph sense, transformations, all through modelling motion. These lines might be followed at different school levels subjected to suitable task design (for example, we are going to carry out similar activities with grade 4, grade 7 and grade 10 students). The software allows for working with graphs of many different types. It leverages two remote controllers of the Nintendo Wii to detect and graphically display the location of two users as they move along lifesize number lines. Embodied interactions with the software are the ground for gaining insights into temporo-spatial mathematical relationships and covariational reasoning. We will discuss these aspects in relation to task design.

Keywords: Functional relationships; Graphs; Movement; Time; Task design

A BODILY APPROACH TO FUNCTIONS

Graphing motion activities have been largely investigated in mathematics education research since the 90s, through the use of motion sensor and other technology (e.g. Nemirovsky et al. 1998; Yerushalmy & Shternberg, 2005; Radford, 2009). Researchers have been studying the ways in which the interaction with this kind of tools may stimulate mathematical thinking while taking advantage of perceptuomotor activity. Even though different researchers have had different conceptions of function, these studies generally share the vision of covariation as a foundation for function in mathematics (see Thompson & Carlson, 2017). Our focus here is on highlighting features of graphing motion activities with a specific technology: a new software application, named WiiGraph [1]. This technology allows for the creation of different types of graphs while two users are moving each a controller of the Nintendo Wii (Wii Remote or Wiimote). Initially, we used it with the idea of introducing secondary school students to variational and covariational reasoning. Drawing on Nemirovsky and colleagues (2013), WiiGraph is a mathematical instrument, that is, "a material and semiotic device together with a set of embodied practices that enable the user to produce, transform, or elaborate on expressive forms (e.g., graphs, equations, diagrams, or mathematical talk) that are acknowledged within the culture of mathematics." (p. 376). Implicating movements of the controllers by two people in an interaction space, activities with WiiGraph also implicate bodily proprioceptive and kinaesthetic experiences both with the devices in use and with the graphical lines and symbolic operations provided by the technology. Nemirovsky et al. (2013) unfold the powerful idea of mathematical instrument to speak about fluent use and mathematical expertise as inseparable from perceptual and motor aspects implied in the activity with the tool. While these researchers are interested in studying fluency with the instrument in the informal context of a scientific exhibition, we focus on the more formal context of the mathematics classroom. In the design of tasks and intervention that we propose in the workshop, the vision of Nemirovsky and colleagues helped us, as researchers and educators, to draw attention to the kind of engagement and practices that activity with the technology might favour within the classroom (e.g., strategic thinking, competitive and collaborative dynamics, use of material resources, etc.; see e.g. Ferrara & Ferrari, 2015a; Ferrara & Ferrari, 2016). We centre on these aspects as a way of discussing challenging lines of flight on covariation, function and families of functions and the issue of designing activities for students from the early years to secondary school. Attending closely to the perceptual and motoric aspects of the experiences, we are interested in offering insights into the ways in which creating and thinking of graphs and functions might change through these experiences and into the new meanings that might emerge from the activities.

WIIGRAPH

To the aim stated above, we focus here on two main options of WiiGraph: *Line* and *Versus*. *Line* furnishes in real time two position-time graphs that capture the distance of the Wiimotes over time from a sensor (origin of the reference system). Time and spatial ranges can be set and modified for the Cartesian axes. This in turn implies specific time interval for the motion to be performed (e.g. 30 seconds) and space constraints for the two users' movements (e.g. at most 10 feet far from the sensor) within the interaction space. Labelled a and b the two distances, the software displays the lines $a(t)$ and $b(t)$ differently coloured on the screen (Figure 1 left). Additionally, selecting the *Make your own Maze!* modality allows for the creation of a target maze to be traversed through the movements. The maze can be built choosing a number of inflection points, a certain value for its thickness and tension, thus a particular graphical arrangement for the maze, which appears on the screen as a tick light blue line. At the end of the session, each user gets a score based on the traversal rate of the created graph with respect to the maze.

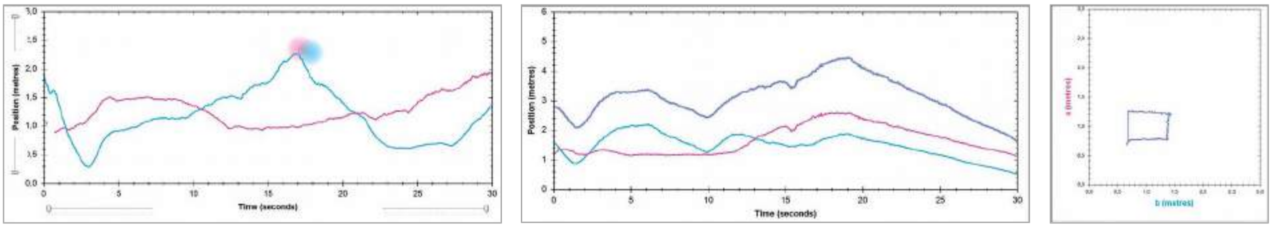

Figure 1. *Line* graphs; *Line* graphs and a+b operation; a rectangle in *Versus*

Within the *Operation* modality, a third coloured graph is shown on the screen: in particular, the addition a+b implies a third graph of position over time that depicts in real time $a(t)$+$b(t)$ (Figure 1 middle); in a word, the graph shows instant by instant the sum of the two distances. It is also possible to choose among the other simple mathematical operations (subtraction/multiplication/division), with an analogous result (a third graph with the chosen characteristics).

The second option we draw attention to is *Versus*, which allows for the creation of a single graph on a Cartesian plane with isometric axes, depending on both users' movement. *Versus* plots at each time t an ordered pair of the positions of the two controllers, showing the line $b(a)$ and leaving time implicit, therefore giving a motion trajectory. Practically, (Figure 1 right) vertical displacement in the graph corresponds to one user's movement, horizontal displacement to the other user's movement.

ACTIVITIES OF GRAPHING MOTION(S): MOVING, COMPARING, TRANSFORMING

In this section, we discuss insights coming from activities with WiiGraph that we carried out through some teaching experiments in Italian classrooms during the last three years. While we recognize that the use of WiiGraph engenders mathematical discourse similar to work with other motion detectors—which have been explored in the literature, we are interested in the ways in which we can exploit the potential of WiiGraph through the design of tasks. We believe that this technology might permit novel reasoning about variation and covariation in the context of graphing motion, therefore new ways of exploring mathematical relationships. In fact, the software requires that two people move in the same interaction space, in the same time interval. In the meanwhile, there are at least two graphs on the screen, which "move" together while originating in real time on the same Cartesian plane. When two students move with the devices, relationships between movements are captured through the relationships between the graphs that are created. Therefore, we can think of activities as (mainly) unfolding along two dimensions. One dimension is concerned with types of bodily engagement of the users with the technology, the other dimension regards how the concepts of graph and function can be grounded on aspects of covariation, coordination and plane transformation. In particular, the ideas shown in the following arise from five different teaching experiment that involved classes of grade 4, 9, 7 and 10 students.

Line and *Versus*

In a first exploration phase, using *Line* option, students might be simply asked to move and make conjectures about the meaning of the lines created on the screen. They might be challenged to move in order to get a couple of graphs with specific shape (like two straight lines or two curved lines). But, profiting of the potential of having two graphs on a single Cartesian plane, they might also be asked to think of ways of moving to obtain two graphs with related shapes (like two parallel slanted straight lines, two meeting straight lines or two translated gibbous lines). Thinking of two parallel straight lines in terms of vertical translation of one to obtain the other, for example, opens room for discussing relationships among the two graphs from a qualitative point of view. This also offers occasions for exploring bodily ways of moving that express given constraints (like parallelism and straightness): for instance, pairs of students have actually been asked to find ways of coordinating together to get the parallel straight lines. While this is rather trivial in the case of horizontal straight lines, it is not in the case of slanted straight lines, in which the two students have to try to keep the same pace. In our experiences, we observed some students drawing attention to each other, in order to maintain their relative positions while moving; some others instead held their hands to keep stretched arms (and fixed distance between them; Figure 2). Different coordination strategies embodied the need of preserving distance among the users to achieve the desired configuration on screen. Students involved in such explorations can give kinaesthetic definitions to vertical translation of graphs, whose mathematical counterpart is the idea of a constant vector that describes a rigid motion.

Figure 2. Bodily movements to capture graphical translation

Using the *Make Your Own Maze!* modality, a maze is added to the graphical space offering a visible shape as the goal of graphing motion for the students. In this case, learners might be challenged to move in pairs to traverse the maze as precise as possible, eventually engaging them in competitive interactions. Being in the challenge means to focus on both quantitative and qualitative aspects of the graphical notation, which are relevant to pursue the highest score. In addition, students might be asked to think of difficult graphs for their mates to match, and to describe features in terms of changes in direction, speed and position. In previous case studies (e.g. Ferrara & Ferrari, 2015b), we have seen how the challenging situations offered by the *Make Your Own Maze!* actually involve degrees of covert/overt coordination among the students. The ways that bodies partake in the creation of the graphs do have a role in the perception and thinking of speeds and shapes.

Selecting the *Operation* modality, for example working with the sum, the students have to shift attention to the relationships among the three graphs on the screen. Beyond this, it is challenging for the students to think of different bodily ways of producing a given sum—imagined or made present through the *Make your own Maze!* modality—like an horizontal straight line. Of course, an horizontal straight line can easily be obtained summing up two suitable horizontal straight lines, however one can discover and discuss further possibilities by summing up pairs of suitable slanted straight lines, etc. Another intriguing experience is that of moving keeping fixed distance (like in the case of two parallel slanted straight lines) and discovering that the sum does not preserve slope. Again, learners enter the realm of collaborative interactions, looking for suitable coordination between their movements.

New possibilities in terms of collaboration and coordinated movement are given by the *Versus* option that displays a single graph. In fact, one of the most interesting challenges for a *Versus* graph involves the creation of plane shapes, like rectangles, rhombuses and circles, or closed lines (see again Figure 1 right). Different ways of bodily coordination are assembled in different graphical lines with specific qualities: for example, moving at the same pace in the same direction produces a piece of line segment slanted by 45 degrees. All of this makes room for focusing on the crucial role of time as independent variable, as well as connecting the spatial relationships in *Versus*, which essentially are motion trajectories, with the space-time relationships in *Line*, which are functional relationships. The kinaesthetic ways of interacting with this option implicate an extended perception of movement, which goes beyond the perception of each student's movement to incorporate the composition of the coordinated movements of the students (de Freitas *et al*., 2017).

Notes

1. WiiGraph has been developed by R. Nemirovsky (Manchester Metropolitan University) and some colleagues (C. Bryant, M. Meloney, B. Rhodehamel) from the Center of Research in Mathematics and Science Education of San Diego State University.

REFERENCES

de Freitas, E., Ferrara, F. & Ferrari, G. (2017). The coordinated movement of a learning assemblage: Secondary school students exploring Wii graphing technology. In E. Faggiano, F. Ferrara & A. Montone (Eds.), *Innovation and Technology Enhancing Mathematics Education*. Springer Verlag. (forthcoming)

Ferrara, F. & Ferrari, G. (2015a). Becoming mathematical subjects by playing mathematical instruments: Gibbous lines with WiiGraph. In S. Carreira & N. Amado (Eds.), *Proceedings of the 12th International Conference on Technology in Mathematics Teaching* (pp. 378-386). Faro, Portugal: Faculty of Sciences and Technology of the University of Algarve.

Ferrara, F. & Ferrari, G. (2015b). Being collaborative, being rivals: Playing WiiGraph in the mathematics classroom. In B. Di Paola & C. Sabena (Eds.), Proceedings of the 67th Conference of the Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques, *Quaderni di Ricerca in Didattica (Scienze Matematiche), 25*(2), 461-472. Palermo, Italy: G.R.I.M., Dipartimento di Matematica e Applicazioni. ISSN: 1592-4424. Available at: http://math.unipa.it/~grim/CIEAEM%2067_Proceedings_QRDM_Issue%2025,%20Suppl.2_WG3B.pdf

Ferrara, F. & Ferrari, G. (2016). Traversing Mathematical Places. In Csíkos, C., Rausch, A., & Szitányi, J. (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, 251-258. Szeged, Hungary: PME.

Nemirovsky, R., Tierney, C. & Wright, T. (1998). Body motion and graphing. *Cognition and Instruction, 16*(2), 119-172.

Nemirovsky, R., Kelton, M.L. & Rhodehamel, B. (2013). Playing mathematical instruments: Emerging perceptuomotor integration with an interactive mathematics exhibit. *Journal for Research in Mathematics Education, 44*(2), 372–415. doi: 10.5951/jresematheduc.44.2.0372

Radford, L. (2009) Signifying relative motion: Time, space and the semiotics of Cartesian graphs. In W.M. Roth (Ed.), *Mathematical Representations at the Interface of the Body and Culture* (pp. 45-69). Charlotte, NC: Information Age Publishers.

Yerushalmy, M. & Shternberg, B. (2005). Chapter 3: Epistemological and cognitive aspects of time: A tool perspective. *Journal for Research in Mathematics Education, Monograph, 13*. Retrieved from http://www.jstor.org/stable/30037731